MATH 1050 Chapter 5 Review

5.1

(8) $\begin{cases} 3x+4y=4 \\ x-y=13 \end{cases} \Rightarrow x = y+13$

$3(y+13)+4y=4$

$3y+39+4y=4$

$7y=-35$

$y=-5$

$x=-5+13$

$x=8$

$(8,-5)$

(29) $\begin{cases} \frac{2x-1}{3}+\frac{y+2}{4}=4 \\ \frac{x+3}{2}-\frac{x-y}{3}=3 \end{cases}$

$(12) \Rightarrow 8x-4+3y+6=48 \rightarrow 8x+3y=46$ (2) $16x+6y=92$

$(6) \Rightarrow 3x+9-2x+2y=18 \rightarrow x+2y=9$ (-3) $-3x-6y=-27$

$13x=65$

$x=5, y=2$

$(5,2)$

(31) $\begin{cases} 9x-5y=1 \\ -18x+10y=1 \end{cases}$ (2) $18x-10y=2$

$-18x+10y=1$

$0+0=3$ FALSE $\Rightarrow \emptyset$

(33) $\begin{cases} 4x-y=9 \\ -8x+2y=-18 \end{cases} \rightarrow y=4x-9$

$-8x+2(4x-9)=-18$

$-8x+8x-18=-18$

$-18=-18$ TRUE $\Rightarrow (x, 4x-9)$

Infinitely Many Solutions

(49) $\begin{cases} x+3y+4z=14 & A \\ 2x-3y+2z=10 & B \\ 3x-y+z=9 & C \end{cases}$

Eliminate y Twice

$A+B$: $3x+6z=24 \rightarrow x+2z=8$ D

B) $2x-3y+2z=10$

$-3C)$ $-9x+3y-3z=-27$

$-7x-z=-17$ E

D) $x+2z=8$

$2E)$ $-14x-2z=-34$

$-13x=-26$

$x=2$ (From D) $z=3$ (From C) $y=0$

$(2,0,3)$

(83) $x^2+y^2+ax+by+c=0$

$(-1,3)$: $1+9-a+3b+c=0 \rightarrow -a+3b+c=-10$

$(6,2)$: $36+4+6a+2b+c=0 \rightarrow 6a+2b+c=-40$

$(-2,-4)$: $4+16-2a-4b+c=0 \rightarrow -2a-4b+c=-20$

Solve same as above problem (#49) $\Rightarrow a=-4, b=2, c=-20$

$x^2+y^2-4x+2y-20=0$

5.3

(7) $\begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = 3(-2)-(5)(4)$

$-6-20=-26$

(11) $\begin{vmatrix} -2 & 0 & 1 \\ 1 & 2 & 0 \\ 4 & 2 & 1 \end{vmatrix}$

$M_{21}=\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix}=0-2$ (-2)

$M_{22}=\begin{vmatrix} -2 & 1 \\ 4 & 1 \end{vmatrix}=-2-4$ (-6)

$M_{23}=\begin{vmatrix} -2 & 0 \\ 4 & 2 \end{vmatrix}=-4-0$ (-4)

$C_{21}=2, C_{22}=-6, C_{23}=4$

(39) $\begin{vmatrix} -2 & 0 & 1 \\ -1 & 3 & x \\ 5 & -2 & 0 \end{vmatrix} = 3$

$(0+0+2)-(15+4x+0)=3$

$-4x-13=3$

$-4x=16$

$x=-4$

(63) $\begin{cases} 4x+3y=-7 \\ 2x+3y=-11 \end{cases}$

$D=\begin{vmatrix} 4 & 3 \\ 2 & 3 \end{vmatrix}=6$

$D_x=\begin{vmatrix} -7 & 3 \\ -11 & 3 \end{vmatrix}=12$

$D_y=\begin{vmatrix} 4 & -7 \\ 2 & -11 \end{vmatrix}=-30$

$x=\frac{D_x}{D}$ $y=\frac{D_y}{D}$

$x=\frac{12}{6}=2$

$y=\frac{-30}{6}=-5$

$(2,-5)$

(73) $\begin{cases} 2x-y+4z=-2 \\ 3x+2y-z=-3 \\ x+4y+2z=17 \end{cases}$

$D=\begin{vmatrix} 2 & -1 & 4 \\ 3 & 2 & -1 \\ 1 & 4 & 2 \end{vmatrix}=63$

$D_x=\begin{vmatrix} -2 & -1 & 4 \\ -3 & 2 & -1 \\ 17 & 4 & 2 \end{vmatrix}=-189$ $x=\frac{-189}{63}$

$D_y=\begin{vmatrix} 2 & -2 & 4 \\ 3 & -3 & -1 \\ 1 & 17 & 2 \end{vmatrix}=252$ $y=\frac{252}{63}$

$D_z=\begin{vmatrix} 2 & -1 & -2 \\ 3 & 2 & -3 \\ 1 & 4 & 17 \end{vmatrix}=126$ $z=\frac{126}{63}$

$(-3,4,2)$

5.5 (15) $3x^2+2y^2=5$, $x-y=-2$ → $x=y-2$ → $3(y-2)^2+2y^2=5$

$5y^2-12y+7=0$

$(5y-7)(y-1)=0$

$y=\frac{7}{5}$, $x=-\frac{3}{5}$ | $y=1$, $x=-1$

(17) $x^2+y^2=8$, $x^2-y^2=0$ → $4+y^2=8$, $y^2=4$, $y=\pm 2$

$2x^2=8$, $x^2=4$, $x=\pm 2$

$\{(-2,-2),(-2,2),(2,-2),(2,2)\}$

(36) $3x^2+xy+3y^2=7$ A

$x^2+y^2=2$ B

A) $3x^2+xy+3y^2=7$

-3B) $-3x^2-3y^2=-6$

$xy=1 \Rightarrow y=\frac{1}{x}$

B) $x^2+\left(\frac{1}{x}\right)^2=2$

$x^2-2+\frac{1}{x^2}=0$

$x^4-2x^2+1=0$

$(x^2-1)(x^2-1)=0$

$(x+1)(x-1)(x+1)(x-1)=0$

$x=-1$, $y=-1$ OR $x=1$, $y=1$

(48) $x+y=10$, $x^2-y^2=20$ $\quad (y=10-x)$ → $x^2-(10-x)^2=20$

$20x-100=20$

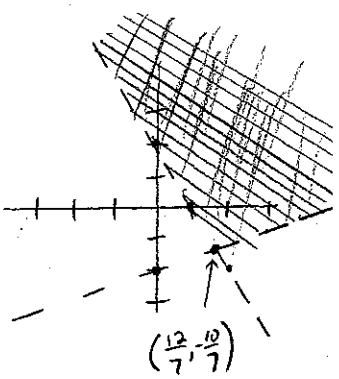
$20x=120$

$x=6$, $y=4$

5.6 (31) $2x+y>2 \Rightarrow y>-2x+2$

$x-3y<6 \Rightarrow y>\frac{1}{3}x-2$

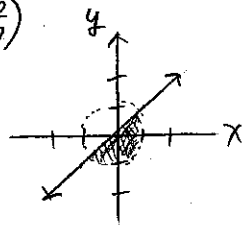
$(\frac{12}{7},-\frac{10}{7})$



$(\frac{12}{7},-\frac{10}{7})$

(34) $y \le x$

$x^2+y^2<1$



5.6 (60) Below $y=-\frac{3}{2}x-3$ (Dashed Line) $\Rightarrow y<-\frac{3}{2}x-3 \Rightarrow 2y<-3x-6 \Rightarrow$ $3x+2y<-6$

Below $y=3x+3$ (Dashed Line) $\Rightarrow y<3x+3 \Rightarrow -3x+y<3 \Rightarrow$ $3x-y>-3$

(69)

(x,y)	$f=3x+5y$	
$(1,1)$	$f=3+5=8$	Minimum
$(6,3)$	$f=18+15=33$	
$(5,10)$	$f=15+50=65$	Maximum
$(2,7)$	$f=6+35=41$	

(77) Let x = # of food boxes

y = # of clothing boxes

CONSTRAINTS

Weight: $40x+10y \le 16{,}000$ $(400,0)+(0,1600)$

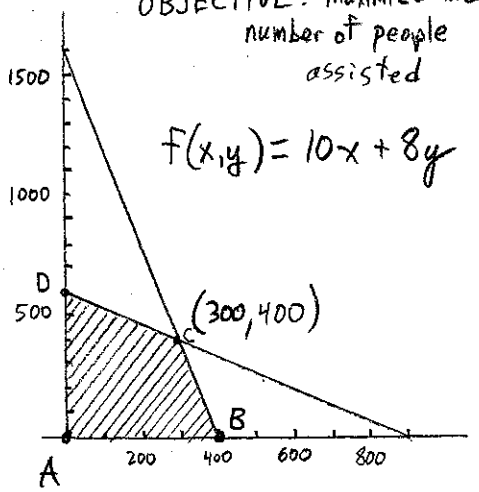
Volume: $20x+30y \le 18{,}000$ $(900,0)+(0,600)$

Common Sense: $x \ge 0$, $y \ge 0$

OBJECTIVE: Maximize the number of people assisted

$f(x,y)=10x+8y$

	(x,y)	$f=10x+8y$	
A	$(0,0)$	$0+0=0$	
B	$(400,0)$	$4000+0=4000$	
C	$(300,400)$	$3000+3200=6200$	Maximum
D	$(0,600)$	$0+4800=4800$	



Summary: 300 Food Boxes & 400 Clothing Boxes will assist a maximum of 6,200 people