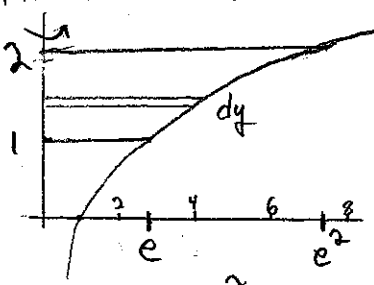
MATH 1220 – Review Problems for Exam 6

6.2 (4)



$$V = \int_1^2 \pi (e^y)^2\,dy = \frac{\pi}{2}\left(e^4 - e^2\right)$$

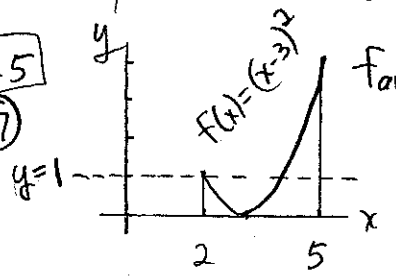
6.6 (13)

$f(y) = 800 + 2y$

$W = \int_0^{500} (800 + 2y)\,dy$

$= \boxed{650{,}000 \text{ Ft-lb}}$

6.5 (7)



$$f_{ave} = \frac{1}{5-2}\int_2^5 \left(x^2 - 6x + 9\right)dx = \boxed{1}$$

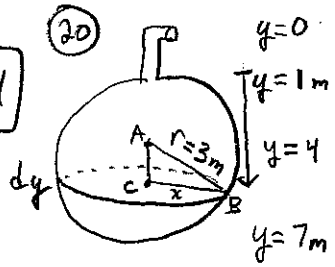
$f(c) = c^2 - 6c + 9 = 1 = f_{ave}$

$c^2 - 6c + 8 = 0$

$(c-2)(c-4) = 0$

$\boxed{c = 2, 4}$

6.6 (20)



$AC^2 + BC^2 = AB^2$

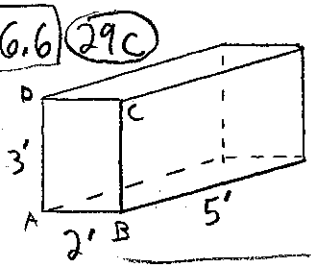
$(y-4)^2 + x^2 = 3^2$

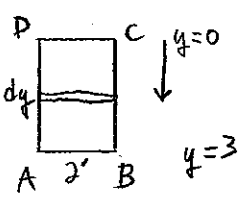
$x^2 = 8y - y^2 - 7$

$W = \int_1^7 [1000(9.8)\pi x^2\,dy]\,y$

$W = 9800\pi \int_1^7 (8y - y^2 - 7)\,y\,dy = \boxed{1{,}411{,}200\pi \text{ Joules}}$

6.6 (29c)





$F = 62.5\int_0^3 (2\,dy)\,y$

$= \boxed{562.5 \text{ lbs}}$

(44)

$m_1 = 6$	$m_2 = 5$	$m_3 = 1$	$m_4 = 4$
$(1,-2)$	$(3,4)$	$(-3,-7)$	$(6,-1)$

$m = \Sigma m_i = 6+5+1+4 = 16$

$M_y = \Sigma m_i x_i = 6(1) + 5(3) + 1(-3) + 4(6) = 42$

$M_x = \Sigma m_i y_i = 6(-2) + 5(4) + 1(-7) + 4(-1) = -3$

$\bar{x} = \frac{M_y}{m} = \frac{42}{16} \quad \bar{y} = \frac{M_x}{m} = \frac{-3}{16}$

CENTROID $\left(\frac{21}{8}, -\frac{3}{16}\right)$

6.7 (8)

$P = 50 - \frac{1}{20}x$

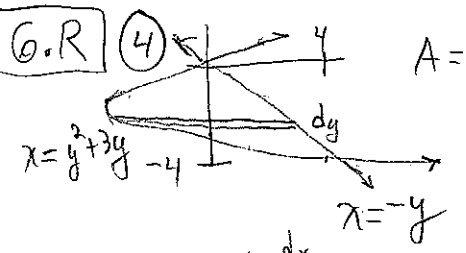
$P_s = 20 + \frac{1}{10}x$

$P = P_s$ at $(200, 40)$ (X, P)

Cons Surp $= \int_0^{200} \left[\left(50 - \frac{1}{20}x\right) - 40\right]dx = \boxed{\$1000}$

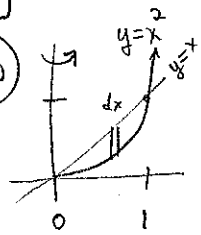
Prod Surp $= \int_0^{200} \left[40 - \left(20 + \frac{1}{10}x\right)\right]dx = \boxed{\$2000}$

6.R (4)



$A = \int_{-4}^{0} \left[-y - (y^2 + 3y)\right]dy = \boxed{\frac{32}{3}}$

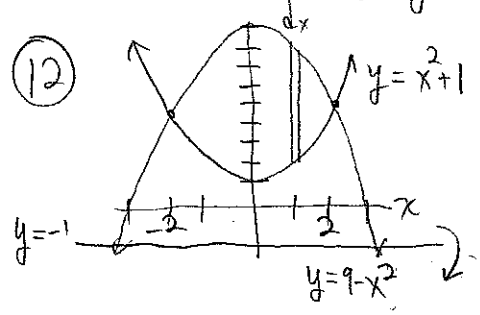
(9b)



$V = 2\pi \int_0^1 x\left(x - x^2\right)dx$

$= \boxed{\frac{\pi}{6}}$

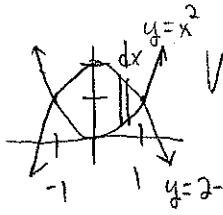
(12)



$V = \pi \int_{-2}^{2} \left[\left(9 - x^2 + 1\right)^2 - \left(x^2 + 1 + 1\right)^2\right]dx$

$= \boxed{256\pi}$

(20)



$V = \int_{-1}^{1} \left[\left(2 - x^2\right) - x^2\right]^2 dx$

$= \frac{64}{15}$

(23) $x = 3t^2, \; y = 2t^3$

$\frac{dx}{dt} = 6t \quad \frac{dy}{dt} = 6t^2$

$0 \le t \le 2$

$L = \int_0^2 \sqrt{(6t)^2 + (6t^2)^2}\,dt =$

$\boxed{10\sqrt{5} - 2}$

(25) $y = \frac{1}{6}\left(x^2 + 4\right)^{3/2} \quad 0 \le x \le 3$

$\frac{dy}{dx} = \frac{1}{2}x\left(x^2 + 4\right)^{1/2}$

$L = \int_0^3 \sqrt{1 + \frac{1}{4}x^2\left(x^2 + 4\right)}\,dx = \boxed{\frac{15}{2}}$

(27) $F(x) = kx$

$30 \text{ N} = k(.03 \text{ m})$

$1000 = k$

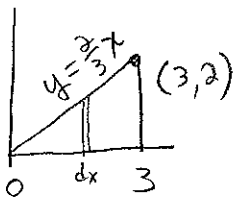
$f(x) = 1000x$

$W = \int_0^{.08} 1000x\,dx =$

$500x^2\Big]_0^{.08} = \boxed{3.2 \text{ J}}$

EXAM 6 (continued)

6.R (32)



$m = \rho \int_0^3 \frac{2}{3}x\,dx = 3\rho$

$M_y = \rho \int_0^3 x \cdot \frac{2}{3}x\,dx = 6\rho$

$M_x = \rho \int_0^3 \frac{1}{2}\left[\frac{2}{3}x\right]^2 dx = 2\rho$

$\bar{x} = \frac{M_y}{m} = \frac{6\rho}{3\rho} = 2$

$\bar{y} = \frac{M_x}{m} = \frac{2\rho}{3\rho} = \frac{2}{3}$

Centroid $\left(2, \frac{2}{3}\right)$

(33) $p = 2000 - .1x - .01x^2 \quad X = 100$

$P = 2000 - .1(100) - .01(100)^2 = 1890$

$\text{Cons Surp} = \int_0^{100} \left[(2000 - .1x - .01x^2) - 1890\right] dx$

$= \$7,166.67$