# MATH 1065 - Review for Chapters 10 and 11

## 10.1

**(26)**
$$\begin{cases} x - y = 5 \\ -3x + 3y = 2 \end{cases}$$

$(3)\to 3x - 3y = 15$

$\longrightarrow -3x + 3y = 2$

$0 + 0 = 17$ (FALSE) → No Solutions ($\varnothing$)

**(47)**
$$\begin{aligned} x - y - z &= 1 & A \\ -x + 2y - 3z &= -4 & B \\ 3x - 2y - 7z &= 0 & C \end{aligned}$$

Let's eliminate $y$ twice:

$B+C$: $2x - 10z = -4 \to x - 5z = -2$ (D)

$2A+B$: $x - 5z = -2$ (E)

$-D+E$: $0+0=0$ (TRUE) Infinite Solutions

From D: $x = 5z-2$ (F)

F into A: $(5z-2) - y - z = 1$

$-y = -4z+3$ or $y = 4z-3$

Answer: $(5z-2,\ 4z-3,\ z)$

Sample Solution to check $(3,1,1)$ or $(-2,-3,0)$

## 10.3

**(33)**
$$\begin{aligned} x + y - z &= 6 \\ 3x - 2y + z &= -5 \\ x + 3y - 2z &= 14 \end{aligned}$$

$$D_y = \begin{vmatrix} 1 & 6 & -1 \\ 3 & -5 & 1 \\ 1 & 14 & -2 \end{vmatrix} = (10+6-42) - (5+14-36) = -26 - (-17) = -9$$

$$D = \begin{vmatrix} 1 & 1 & -1 \\ 3 & -2 & 1 \\ 1 & 3 & -2 \end{vmatrix} = (4+1-9) - (2+3-6) = -4 - (-1) = -3$$

$$y = \frac{D_y}{D} = \frac{-9}{-3} = \boxed{3}$$

$x$ and $z$ can be found from $x = \dfrac{D_x}{D}$ and $z = \dfrac{D_z}{D}$

## 10.5

**(39)** $\dfrac{x^2}{x^3 - 4x^2 + 5x - 2}$ Decomposition $\longrightarrow = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x-2}$

$(x-1)^2(x-2)$

First Factor Denominator

Zeros of Denominators

| | 1 | -4 | 5 | -2 |
|---|---|---|---|---|
| 1 | 1 | -3 | 2 | 0 |
| 1 | 1 | -2 | 0 | |
| 2 | 1 | 0 | | |

$(x-1)^2(x-2)$

$x^2 = A(x-1)(x-2) + B(x-2) + C(x-1)^2$

Choose $x=1$: $1 = 0 - B + 0 \Rightarrow B = -1$

Choose $x=2$: $4 = 0 + 0 + C \Rightarrow C = 4$

$x^2$: $1 = A + C \Rightarrow A = -3$

Answer: $\boxed{\dfrac{-3}{x-1} - \dfrac{1}{(x-1)^2} + \dfrac{4}{x-2}}$

**(46)** $\dfrac{x^2+9}{x^4 - 2x^2 - 8} = \dfrac{A}{x+2} + \dfrac{B}{x-2} + \dfrac{Cx+D}{x^2+2}$

$(x^2-4)(x^2+2)$

$(x+2)(x-2)(x^2+2)$

$x^2 + 9 = A(x-2)(x^2+2) + B(x+2)(x^2+2) + (Cx+D)(x+2)(x-2)$

Choose $x=-2$: $13 = -24A \Rightarrow A = \dfrac{-13}{24}$

Choose $x=2$: $13 = 24B \Rightarrow B = \dfrac{13}{24}$

$x^3$: $0 = A + B + C \Rightarrow C = 0$

Const: $9 = -4A + 4B - 4D \Rightarrow D = -\dfrac{7}{6}$

Answer: $\boxed{-\dfrac{13}{24(x+2)} + \dfrac{13}{24(x-2)} - \dfrac{7}{6(x^2+2)}}$

## 10.6

**(15)**
$$\begin{aligned} y &= 3x - 5 \\ x^2 + y^2 &= 5 \end{aligned}$$

$x^2 + (3x-5)^2 = 5$

$x^2 + 9x^2 - 30x + 25 = 5$

$10x^2 - 30x + 20 = 0$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$\boxed{x=2,\ y=1}$ $\boxed{x=1,\ y=-2}$

**(41)**
$$\begin{aligned} x^2 + 2y^2 &= 16 & A \\ 4x^2 - y^2 &= 24 & B \end{aligned}$$

$A + 2B$: $9x^2 = 64$

$x^2 = \dfrac{64}{9} \Rightarrow$ Into A: $\dfrac{64}{9} + 2y^2 = 16$

$x = \pm\dfrac{8}{3}$

$2y^2 = \dfrac{80}{9}$

$y^2 = \dfrac{40}{9}$

$y = \pm\dfrac{2\sqrt{10}}{3}$

$$\left(\pm\frac{8}{3},\ \pm\frac{2\sqrt{10}}{3}\right)$$

**11.1** (12) $\dfrac{12!}{10!} = \dfrac{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9\cdot10\cdot11\cdot12}{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9\cdot10} = 11\cdot12 = 132$

(24) $\{a_n\} = \left\{\dfrac{3^n}{n}\right\}$ $a_1 = \frac{3}{1}$, $a_2 = 9/2$, $a_3 = 27/3$, $a_4 = 81/4$, $a_5 = 243/5$

(28) $\dfrac{1}{1\cdot2}, \dfrac{1}{2\cdot3}, \dfrac{1}{3\cdot4}, \dfrac{1}{4\cdot5}, \ldots$

$n=1 \quad n=2 \quad n=3 \quad n=4$

$\boxed{\{a_n\} = \dfrac{1}{n\cdot(n+1)}}$

(76) $\displaystyle\sum_{k=0}^{14}(k^2-4) = -4-3+0+5+12+21+32+45+60+77+96+117+140+165+192$

$= \boxed{955}$

**11.2** (25) $1, -2, -5, \ldots$ Find $90^{TH}$ Term

$a_1 = 1 \quad d = -3 \quad a_{90} = 1+(90-1)(-3) = \boxed{-266}$

(31) $a_9 = -5, a_{15} = 31$

$15-9=6 \quad 6d = 31-(-5)$ $\boxed{d=6}$

$a_1 = a_9 - 8d, \quad a_1 = -5-48$

$\boxed{a_1 = -53}$

Recursive: $\boxed{a_n = a_{n-1} + 6 \ (a_1 = -53)}$

$n^{TH}$ Term: $\boxed{a_n = -53 + (n-1)6}$

OR $a_n = 6n - 59$

**11.3** (28) $1, 3, 9, \ldots$ Find $8^{TH}$ Term

$r = 3 \quad a_8 = 1(3)^{8-1} = \boxed{2187}$

(50) $\displaystyle\sum_{n=1}^{15} 4\cdot3^{n-1}$ $(r=3)$

$= 4+12+36+108+\cdots 19{,}131{,}876$

$S_{15} = \dfrac{4[1-3^{15}]}{1-3} = \boxed{28{,}697{,}812}$

(57) $2 - \frac{1}{2} + \frac{1}{8} - \frac{1}{32} + \cdots$ $(r = -\frac{1}{4})$

$S_\infty = \dfrac{2}{1-{-\frac{1}{4}}} = \dfrac{2}{5/4} = \dfrac{8}{5}$

**11.5** (22) $(2x+3)^5$

$= 1(2x)^5(3)^0 + 5(2x)^4(3)^1 + 10(2x)^3(3)^2 + 10(2x)^2(3)^3 + 5(2x)^1(3)^4 + 1(2x)^0(3)^5$

$= 1(32x^5) + 5(16x^4)3 + 10(8x^3)9 + 10(4x^2)27 + 5(2x)81 + 1(1)243$

$= \boxed{32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243}$

(33) $(2x+3)^9$ Find the $x^7$ coefficient

$\binom{9}{2}(2x)^7(3)^2 = 36(128x^7)9 = \underline{41472}\,x^7$

(37) $(3x-2)^9$ Find the $3^{RD}$ Term

$\binom{9}{2}(3x)^7(-2)^2 = 36(2187x^7)4 = \boxed{314928x^7}$